AN INTEGRATED MODEL OF A
REPRESENTATIVE DEFENSE CONTRACTOR

David Robert Clark

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AN INTEGRATED MODEL OF A
REPRESENTATIVE DEFENSE CONTRACTOR

by

David Robert Clark

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Thesis Advisor: C.R. Jones

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(20. ABSTRACT Continued)

segment describes the output and contractual relationships of the buyer and the seller. The financial segment relates the balance sheet and the income statement to the structure of the firm. A discussion of decision rules for the behavior of the defense contractor operating at equilibrium is conducted.

An Integrated Model of a
Representative Defense Contractor

by

David R. Clark
Captain, United States Army
B.S., Gannon College, Erie, Pennsylvania, 1968

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TABLE OF SYMBOLS

A	cash
Ap	accounts payable
Ar	accounts receivable
B	bonds — long term debt
C	cost contract
D	debt — short term
E	equity
F	fixed cost
G	government furnished equipment
H	multi-product output function
I	input to inventory
J	capital maintenance
K	capital
L	labor
M	raw material
N	earnings after taxes
P	price in dollars
Q	quantity output
R	retained earnings
S	subcontracted parts
W	withdrawn from inventory
X	inventory
Z	final contract cost settlements
b	unit value of bonds
d	unit value of debt

e	market price of owners equity
f	rate of return on equity
i	number of a commercial contract
j	number of a government contract
$*$	a contract type i or j as appropriate
ℓ	number of an incentive
r	tax rate
t	time
x	last number of a commercial contract
y	last number of a government contract
z	last performance specification
B	value of long term debt
CGS	cost of goods sold
D	value of short term debt
E	value of equity to the firm
EBIT	earnings before interest and taxes
EBT	earnings before taxes
GP	gross profit
K	value of capital to the firm
R	revenue
α	target profit rate
β	incentive profit rate for reduced cost
γ	incentive profit rate for performance
Γ	performance specification
δ	depreciation rate of capital
Δ	fractional change in a variable
ε	incentive profit rate for delivery

θ	interest rate on debt
ξ	progress payment rate
π	profit
ρ	coupon rate on bonds
ϕ	portion of repayable debt

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I. INTRODUCTION

A. BACKGROUND

1. Need for the Model

In the last five years all the levels of government, from the Congress through the Armed Sercives, have taken interest in the procurement process and its problems. This interest is expressed in reports of activities such as "The Commission on Government Procurement" (GOGP) sponsored by the Congress [5], "The Industry Advisory Council (IAC) Subcommittee: To Consider Defense Industry Contract Financing," sponsored by the Secretary of Defense [12], and "The Materiel Acquisition Review Committees," sponsored by each service. In particular the Army Materiel Acquisition Review was of interest. Each of these committee reports looks at the procurement process from a different point of view and level within government. Each report at its own level seeks to analyze the procurement process. All of these reports and analyses finally boil down to considerations between the buyer (the government) and the seller (the Defense Contractor).

The government motivations and procurement views have received much attention in these reports. Many new procurement policies and procedures are being formulated for implementation in the procurement areas. As these new policies are implemented, questions like the following are being asked:

"What factors in the general economy will these new procedures effect?" "How will the defense contractors and subcontractors have to adjust their operations to comply with these new policies?" Few answers if any are available to these questions. In the financial area, for example, the IAC Subcommittee Report says,

> ... the only tool available to measure the effects of change in financing parameters is the Air Force Financing Model. It would not be feasible to use that model for evaluating different financing packages for the myriad contracts negotiated every year.[1]

The model developed in this thesis is an effort in basic research. The modeling effort is directed towards developing a tool with which the government can analyze the effects of its procurement policies on the defense industry and, in turn, the whole economy.

2. Literature Review

The modeling effort in the current literature combines techniques in four areas: mathematical modeling, utility theory, expected value theory, and maximization theory. A common technique is used in applying the maximization theory, expected value theory and utility theory. The mathematical model is used to establish a measure of success by which the firm can guide its decisions. Profit, net income, or a firm's wealth are candidate measures of success. Utility theory is used to describe the decision makers preferences for a range

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[1]Industry Advisory Council Subcommittee, To Consider Defense Industry Contract Financing, p. 20, June 1971.

of values for the measure of success. Expected value theory permits the introduction of uncertain outcomes of the measure of success. Maximization theory permits the characterization of optimal choices, given the measure of success, the decision makers preferences and a distribution of uncertain outcomes.

For further discussion of optimization under uncertainty see References 2, 13, 18, and 19.

The mathematical formulation of the measures of success of Baron, Leland, Sandmo and Vickers are discussed in the following paragraph.

Baron [2] takes total wealth as his measure of success which is to be optimized under the conditions of risk and uncertainty discussed above. Baron also formulates an incentive type profit function for his model. This incentive profit function is utilized in this thesis.

Leland [13] chooses profit as the measure of success for his mathematical model. He calculates profit simply as revenue minus cost. The thrust of his work is toward analyzing the effects of risk aversion on the firm.

Sandmo [18] chooses a similar model to Leland. Fixed and variable costs are modeled separately to determine the effects each has on the analysis. Sandmo, like Leland, analyzes his model to determine the effects of risk aversion on the firm. Leland and Sandmo, though working independently, produce theoretical contributions in the same areas. For those interested in further research on risk and uncertainty refer to Cummings [6].

Vickers [19] establishes net income as the measure of success in his model of the firm. He develops a complicated model which considers revenue, fixed and variable costs, the firm's wealth and input and output factors. Vicker's model is unique in that his formulation integrates input factors, output factors and financial considerations.

The model in this thesis adapts the general modeling techniques described above to the defense contractor. The defense contractor model advances the current modeling efforts through modeling inter-market effects for output market and the financial market on the decision process of the firm.

In Chapter I, Section B, of this paper, the general characterization of the defense contractor model is presented. Chapter II gives a detailed analysis of the model characteristics, the supply of input market, the manufacturing process, the demand for output market, and the financial market. An analysis of decision rules at the optimum is presented in Chapter III. A summary and areas for future research are presented in Chapter IV.

B. CHARACTERIZATION OF THE MODEL

The model presented in this thesis is designed to characterize the interactions of a defense contractor's management, which deals with the functions of manufacturing, finance, supply of input and demand for output. Figure I schematically depicts the fact that the four functional area of input or supply, output or demand, finance and manufacturing

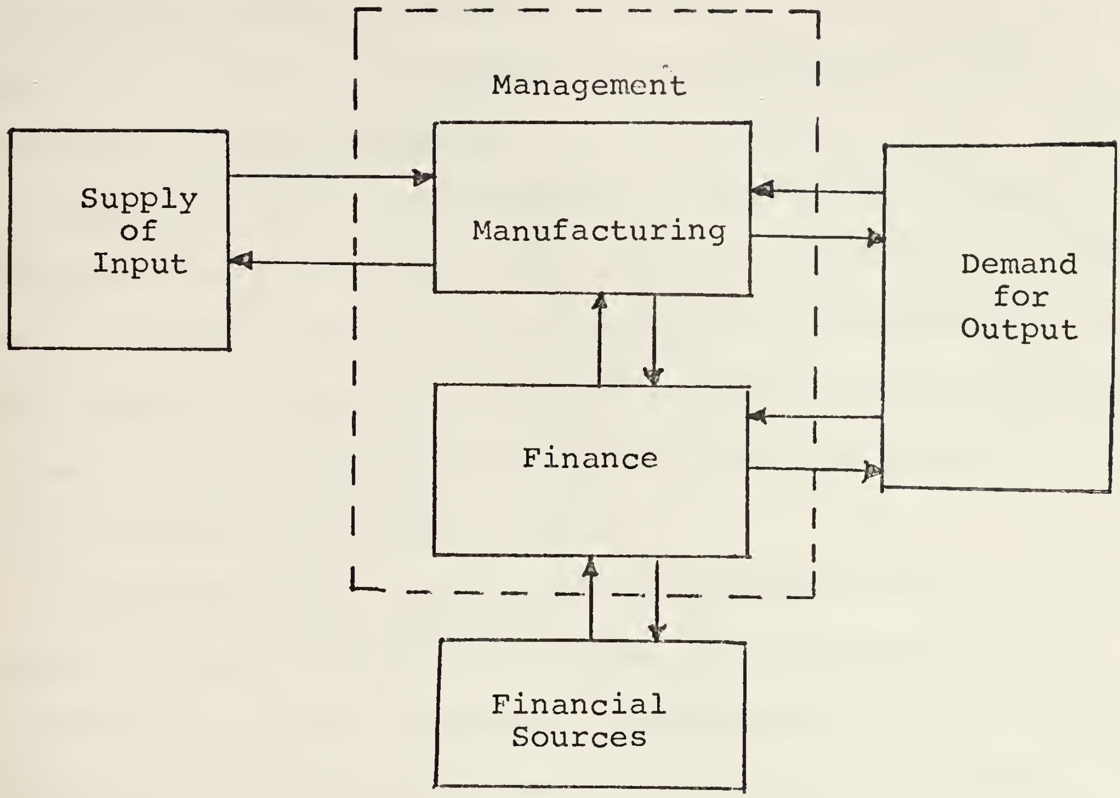

FIGURE 1. Defense Contractor

interact with each other in a tangible, definable manner. Figure 1 further portrays the role of management. The functional area of management is incorporated into the manufacturing and financing relationships directly. The decision aspects of management are embodied in the model as the decision variables whose optimal choice characterize the defense contractor from period to period.

The supply segment of the model is viewed in the context of pure and perfect competition scenario. Raw materials, labor and capital are available at predetermined prices exogenous to the contractor.

Manufacturing is represented as a two-part process, inventory and production. This view of manufacturing allows an uncluttered view of the interaction of manufacturing with the supply of outputs, the demand for inputs and the financial aspects. Yet, it is presented with enough complexity so as not to render the process trivial.

The demand for output by the government and by the public sector is approached from the view of a contract document. Government contracts can be of a negotiated type between the government and the contractor, or a bid award type where the low responsible bidder is awarded the contract. This interaction is conducted over a range of buyer-seller relationships, such as imperfectly competitive, monopsonistic or bilateral monopoly.

The defense contractor's activities in the commercial demand market are considered strictly on a price and quantity basis. When a defense contractor receives a commercial contract, this model considers only the quantity to be produced and the unit selling price for that contract.

The finance segment considers interactions between manufacturing, supply of input, demand for output, and the financial market. Internally to the firm, alternative inventory and production possibilities are traded off against

the firm's financial structure. Externally to the firm, the government, through its contracting policies and the financial market, establishes financial boundaries, influences interest rates, and the value of owners equity, through market interactions. With these constraints established, the defense contractor adjusts his financial position to accomplish his production goals.

II. EXPOSITION OF THE MODEL

A. MODEL CHARACTERISTICS

Some preliminary remarks concerning the treatment of time as a discrete variable and the nature of the model notation will facilitate understanding the formulation of the model.

Time

Time is a discrete variable in this model. A discrete formulation is appropriate since phenomena associated with the model occur in discrete time increments. Contracts and a firm's accounting periods are examples of discrete time phenomena.

Notation

It is observed that variables can be placed into collective classes. For example, the class of raw materials for production is represented by the variable M. The collective unit price for raw materials is written $\bar{P}^M$. This notation allows a representative firm in a general industry to be modeled.

Upon application of this model to a representative firm in a specific industry, the class variables must be replaced by detailed accounting data.

Symbols

A table of symbols is included on page 7. To avoid ambiguity each variable class is represented by only one variable symbol throughout this thesis.

Exogenous Terms

All symbols appearing in this thesis with a bar ($\bar{\ }$) over it, e.g., $\bar{a}$, are considered to be specified outside the model.

Endogenous Terms

All symbols appearing without a bar are considered to have been derived or calculated from variables internal to the model. Endogenous terms represent decision variables or are calculated terms resultant from other management decisions.

Variable Interpretation

A standard notation is used for all variable descriptions, e.g.,

$$\text{Disposition} \rightarrow W_{M}{}^{i}_{t} \begin{array}{l} \leftarrow \text{contract type and number} \\ \leftarrow \text{time period} \end{array}$$

$$\uparrow \text{principle variable}$$

In the example the notation should be read as "the amount of raw material, M, withdrawn from inventory (W) used in a private contract, i, during a time period, t. A principle variable may be used with or without modifying descriptors.

Price Interpretation

All prices contain the principle symbol P and are modified by appropriate descriptors, e.g.,

$$P^{s}_{t} \begin{array}{l} \leftarrow \text{commodity} \\ \leftarrow \text{time period} \end{array}$$

In the example the notation should be read as "the price, P, of subcontracted parts, s, in period, t. Prices always appear with two modifying descriptors, one designating the time period and the other the commodity for which the price applies.

Each segment of the model, supply of input, manufacturing, demand for output and financing is explained in turn in the following sections.

B. SUPPLY OF INPUT

The supply of input assumes a set of pure and perfect markets. This means that the quantity of input purchased will not affect the market price of that input. The following supply of input variables are considered: capital, K, labor, L, raw materials, M, and subcontracted parts, S. The later, subcontracted parts (S), is differentiated from other raw materials to point out the effect government contracts have on subcontractors who work with primary government contractors. The price of subcontracted parts is determined through the interaction of the primary contractor and subcontractor.

Capital

The term capital as used in this thesis refers to plant and equipment. The plant consists of buildings and land used by the contractor. Equipment is composed of tools, trucks, machines and the like. The class variable K which represents capital does not differentiate between plant and equipment.

Capital is treated as a homogeneous variable. The equations discribing capital are written as follows:

$$(1) \qquad K_t = K_{t-1} + K_t - \delta K_t$$

$$(2) \qquad K_t = {}^{M}K_t + {}^{S}K_t + {}^{H}K_t$$

Equation 1 should be read as "the capital available to the contractor in time period t is equal to the capital available in the previous period (t-1), plus the amount of capital purchased during period t, minus the physical depreciation of contractor capital during period t." Equation 2 accounts for the utilization of the available capital during period t. Equation 2 should be read as "the capital available in period t is equal to the sum of the capital devoted to raw material inventory, plus that devoted to the subcontracted parts inventory, plus that devoted to production.

Labor

Labor is a service purchased from period to period. Labor is also considered to be homogeneous throughout this thesis. The equation accounting for the cases of labor of the contractor follows:

$$(3) \qquad L_t = {}^{M}L_t + {}^{S}L_t + {}^{H}L_t$$

Equation 3 should be read as "the total labor in time period t is equal to the labor devoted to raw material in period t plus the labor devoted to subcontracted parts in period t, plus the labor devoted to production in period t."

Raw Materials and Subcontracted Parts

The supply of input equations for raw materials and subcontracted parts are identical in form.

$$(4) \qquad M_t = M_{t-1} + {}^{I}M_t - {}^{W}M_t$$

$$(5) \qquad S_t = S_{t-1} + {}^{I}S_t - {}^{W}S_t$$

Equation 4 can be read as "the quantity of raw materials available to the contractor in the time period t is equal to the quantity of material available at the beginning of the previous period t, plus the material added to inventory in period t, minus the amount of material withdrawn from the inventory in period t." Equation 5 for subcontracted parts is read the same as equation 4 for raw materials. By replacing the word materials in equation 4 with the words subcontracted parts, equation 5's meaning can be interpreted.

Government Furnished Equipment (GFE)

GFE is considered as a special supply of input available for use only in government contracts. As with capital, GFE normally is composed of special items of material, plant and equipment. In this thesis GFE is considered a homogeneous variable. When GFE is utilized in a contract, the contractor

pays a nominal charge $\bar{P}^G_t$ for each unit of GFE. For example, this would represent handling charges.

C. MANUFACTURING SEGMENT

Manufacturing is treated at an aggregate level. That is it is only subdivided into inventory and production.

Inventory

The inventory structure is defined by two sets of equations identical except for the principle variable. The equations are:

$$(6) \quad M_t \leq {}^{M}K_t \qquad\qquad (8) \quad S_t \leq {}^{S}K_t$$

$$(7) \quad {}^{W}M_t = M({}^{M}L_t, {}^{M}K_t) \qquad\qquad (9) \quad {}^{W}S_t = S({}^{S}L_t, {}^{S}K_t)$$

Equation 6 can be read as "the raw materials available from inventory in period t is less than or equal to the 'warehouse' capacity, that is, capital k devoted to material storage in period t." Equation 7 can be read as "the amount of raw materials withdrawn from inventory for use in production in period t is equal to some function M of the labor devoted to material inventory handling and the capital devoted to material inventory handling." Equations 8 and 9 require parallel interpretations of subcontracted parts as equations 6 and 7 do for raw materials.

It should be noted at this point that taken together, equations 4, 6, and 7 link together actions from the supply of input market, the inventory process and the production process relating to raw materials. Equations 5, 7, and 9 do the same for subcontracted parts. This type of a linkage thread, when established for the variables considered in this thesis, interlock to weave the fabric of a unified model.

Production

The production process is modeled as:

$$(10) \qquad H^*(Q_t^*, \Gamma_\ell^*, M_t^*, S_t^*, {}^H L_t^*, {}^H K_t^*, G_t^*) = 0$$

Equation 10 can be read as "there is some implicit production function H* which relates the input variables M_t^*, S_t^*, ${}^H L_t^*$, ${}^H K_t^*$, G_t^* and output variables Q_t^*, Γ_ℓ^* in each period." Further elaboration on the nature of the production function is necessary here. The model of the production process is a multi-product output model. The output for each contract * in time period t is a vector. It is composed of Q_t^*, Γ_ℓ^*, $\ell = 1,\ldots,z$. Q_t^* should be read as "the amount of product Q produced in period t under contract *." Γ_ℓ^* should be read as "a performance characteristic attributed to Q_t^*." The input variables are decision variables for the firm. As trade-offs are made between these variables in a specific production process, the values of the output vector of that

product change. Since government contracts are written with product quantity and minimum performance specifications, input trade-offs are important because they directly affect profit.

D. DEMAND FOR OUTPUT

The demand for output is divided into commercial demand and government demand.

Commercial Contracts

Commercial contracts comprise a substantial percentage of some defense contractors work. Recognizing this fact, two aspects of commercial contracts are described in this thesis. The first apsect is modeled as revenue from commercial contracts written $\sum_i P_t^i Q_t^i$ which can be read as "the sum over all commercial contracts i of the price P_t^i charged per unit output in time period t, times the quantity of product Q_t^i produced and delivered under contract i in time period t." The second aspect considered is the cost of commercial products to the contractor. These costs are accounted for through the expression:

$$(11) \qquad \sum_i \bar{P}_t^M W_{M_t}^i + \bar{P}_t^S W_{S_t}^i + \bar{P}_t^L L_t^i + \bar{P}_t^J K_t^i + \bar{P}_t^K K_t^i$$

Equation 11 can be read as "the cost in period t of all commercial contracts is the sum over all i contracts in time period t of the price, times the quantity of raw materials used in each contract, plus the price times the amount of

subcontracted parts in each contract, plus the price times the labor used in each contract, plus the operating price times the capital used in each contract, plus the cost of capital times the depreciation rate times the capital used in each contract."

Government Contracts

The prime motivation of this work is to be able to forecast contractors responses to changes in procurement policy. To this end, government contract considerations are modeled in detail. The aspect of government contracts relevant to this model and of the most concern from a contractors point of view are: the quantity of product to be produced, the cost, the performance, the schedule goals, the method by which contractor costs will be reimbursed and the method of calculation and payment of profit.

Progress Payments and Cost Reimbursements

For contracts of more than one period in duration, contractor costs are reimbursed through progress payments periodically and through a final settlement upon completion of a contract. The equation which describes progress payments is written:

$$(12) \quad Pr_t^j = \bar{\xi}^j \sum_{t=1}^{t-1} C_t^j - \sum_{t=1}^{t-2} Pr_t^j$$

Equation 12 can be read as "the progress payment made on contract j during period t is equal to a maximum or allowable percentage ξ^j of the cost of performing contract j, up to period t-1, minus the sum of all previous progress payments." Upon completion of the contract a final settlement is made. The equation representing final settlement is written:

$$(13) \qquad Z_T^j = \sum_{t=1}^{T} C_t^j - \sum_{t=1}^{T} \cdot Pr_t^j$$

Equation 13 can be read as "the final cost reimbursement Z on contract j is equal to the total cost of contract j minus the sum of all progress payments." Progress payments directly affect the financial structure of defense contractors. As is shown in the financial section, progress payments contribute to the cash flow of the firm. This cash flow reduces contractor commercial borrowing needs. Contractors are also expected to perform these contracts at a lower cost, since the progress payments are interest free.

Profit

Profit is divided into two classes, normal profit and incentive profit. Normal profit is the compensation received by a firm on a routine basis. Normal profit is calculated as a percentage of the cost of production. Incentive profit is a reward for achieving or surpassing a cost, performance or schedule goal. The profit equation formulated in this thesis

has a term in it which characterizes normal profit and each of the incentive profits. The profit equation is written:

$$(14) \qquad \pi_t^j = \overline{\alpha}^j \, \overline{C}_t^j + \overline{B}^j(\overline{C}_T^j - C_T^j) + \left(\sum_{\ell=1}^{z} \gamma_\ell(\Gamma_\ell - \overline{\Gamma}_\ell)\right)^j$$

$$+ \overline{\varepsilon}\left(\sum_{t=1}^{t} Q_t^j - \sum_{t=1}^{t} \overline{Q}_t^j \right)$$

All the terms in equation 14 need not apply to each government contract. The profit equation is formulated to allow profit payments during each time period in which the contract is in force. Profit payments occur usually at the completion of the contract or upon demonstration of the achievement of some incentive goal. Equation 14 can be read as "the profit in period t on contract j is equal to the normal profit, plus a cost incentive profit, plus the performance incentive profit, plus the schedule incentive profit." Each term is explained in detail below.

<u>Normal Profit</u>

Normal profit, represented by $\overline{\alpha}^j\overline{C}_t^j$, can be read as "the normal profit rate for contract j $\overline{\alpha}^j$, times the target cost for contract j C_t^j. The profit rate is set by the government and the firm during the negotiation phase of contracting. Normal profit is paid at the conclusion of the contract.

Incentive Profit

Incentive profit is the name given to the money paid a contractor upon meeting or exceeding a contract incentive goal. The use of incentive clauses in government contracts is widespread. The technique used here to model incentive profits was adapted from Barron [2]. Each category of incentive profit is discussed below. For further discussions concerning incentive profits see Reference [14].

Cost Incentive Profit

The term $\bar{B}^j(\bar{C}^j_T - C^j_T)$ represents the cost incentive. It can be read as "cost incentive profit is a percentage $\bar{B}^j$ of the cost savings over target cost $(\bar{C}^j_T - C^j_T)$ to be payed the contractor." Note that if actual cost C^j_T is greater than target cost $\bar{C}^j_T$ there is a penalty to the contractor for cost overrun. The contractor in this situation usually absorbs the cost overrun through reduced normal profits.

Performance Incentive Profit

Performance incentives are employed to obtain the best product through innovations in technology or production techniques. The performance incentive term is written:

$$(15) \qquad \left(\sum_{\ell}^{z} \bar{\gamma}_{\ell}(\Gamma_{\ell} - \bar{\Gamma}_{\ell}) \right)^j$$

Term 15 can be read as "the total performance incentive profit on contract j is the sum of the profit from ℓ performance specifications, where each performance specification has a

performance profit rate times a measure of how well the performance is met $(\Gamma_\ell - \bar{\Gamma}_\ell)$." Note the formulation of performance specifications in equation 14 allows for performance shortfalls and subsequent contractor penalties as well as rewards.

Schedule Incentive Profit

The term representing schedule incentive is written:

$$(16) \quad \bar{\varepsilon}^j \left(\sum_{t=1}^{t} Q_t - \sum_{t=1}^{t} \bar{Q}_t \right)^j$$

This term can be read as "the schedule incentive rate ε, times the sum of product produced up to and including time period t on contract j, minus the sum of the scheduled production on contract j up to period t." This formulation provides for rewards and penalty incentives.

E. FINANCIAL SEGMENT

The financial section is discussed through the framework of two accounting concepts, the balance sheet and the income statement. For a detailed discussion of financial statements see Reference [16].

1. Balance Equations

The balance sheet is a stock concept. It lists financial assets of the firm and equates them to the liabilities and net worth of the firm at a point in time. When a series of balance sheets are considered together, a picture of the

changing financial posture of the firm can be observed. These balance sheets, when coupled with their corresponding income statements (discussed below), provide the observer with a chronology of the financial history of the firm. The exposition of the balance sheet and income statement which follow draws on the firm's history to develop financial characteristics.

ASSETS		LIABILITIES AND NET WORTH	
Cash	A_t	Accounts Payable	Ap_t
Accounts receivable	Ar_t	Short-term debt	D_t
Inventory	X_t	Long-term debt	B_t
Plant and equipment	K_t	Owners equity	E_t
Accumulated depreciation	$\Sigma\ \delta K_t$	Retained earnings	R_t
Total Assets	XXXXX	Total Liabilities and Net Worth	XXXXX

Figure 2. BALANCE SHEET

Figure 2 is an idealized example of a balance sheet. It suffices for the purposes of this thesis paper to recognize that the total at the bottom of the left side or asset side of the balance sheet must be equal to the total at the bottom

of the right side or the liabilities and net worth side of the balance sheet. The equation which is a symbolic translation of the balance sheet follows:

$$(17) \qquad A_t + Ar_t + X_t + K_t + \sum_{t=1}^{t} \delta K_t =$$

$$= Ap_t + \mathcal{D}_t + \mathcal{B}_t + \mathcal{E}_t + R_t$$

This equation is called the balance sheet equation. Each term of this equation is discussed in turn below.

<u>Cash</u>

The cash term A_t in the balance sheet equation and on the balance sheet represents only a single entry, cash on hand at the end of the period. The important concept embodied in the cash term is the accounting for the receipts and expenditures that pass through the cash term during the accounting period. These transactions are captured in the cash flow equation:

$$(18) \qquad A_t = A_{t-1} + \sum_j (Pr^j_{t-1} + \pi^j_{t-1} + Z^j_T) + \sum_i P^i_t Q^i_t$$

$$- [\overline{P}^M_{t-2} \, I_{M_{t-2}} + \overline{P}^S_{t-2} \, I_{S_t} + \overline{P}^L_t \, L_t + \overline{\theta} \, D_t$$

$$+ \overline{\rho} \, B_t + \overline{P}^F_t F_t + \overline{P}^K_t \, \Delta K_t + \overline{P}^G_t \, G_t]$$

$$+ \overline{d}_t \, \Delta D_t + \overline{b}_t \, \Delta B_t + \overline{e}_t \, \Delta E_t - \overline{f}_t \, E_t$$

Equation 18 can be read as "the cash available at the end of period t is equal to the cash at the end of period (t-1), plus the cash received from all government contracts in period t (see page 40), plus the cash received from all commercial contracts in the period t (see page 41), minus the cost of raw materials and subcontracted parts received in inventory in period (t-2) (the payment of invoices is delayed two time periods based on the accounts payable policy (page 36)), minus the cost of labor in period t, minus the interest on short term debt in period t, minus the cost of capital purchased in period t, minus the cost of GFE in period t, plus the funds available from assuming short term debt in period t, plus the funds from the sale of owners equity in period t, minus the dividend paid on owners equity in period t."

Accounts Receivable

The equation describing accounts receivable is written:

$$(19) \qquad Ar_t = \sum_j (Pr_t^j + \pi_t^j + Z_T^j) + \sum_i P_t^i Q_t^i$$

Equation 19 can be read as "the accounts receivable in period t is equal to the sum of all funds from government contracts (see page 40), plus the sum of all commercial sales (see page 41). Accounts receivable items of period t are received in period (t+1).

Inventory

Inventory is typical of a class of equations involving gains or losses of value from stock on hand. Capital is the other example considered in this thesis. The accounting scheme chosen for inventory calculations is Last In First Out (LIFO). The cost assigned to stock issued from inventory is the price of the last units that went into inventory [16]. Items of stock purchased in one period are undifferentiable from items purchased in any other period. This accounting process simplifies the inventory modeling problem. The inventory equation is written:

$$(20) \qquad X_t = X_{t-1} + \bar{P}^M_t \, {}^{I}M_t + \bar{P}^S_t \, {}^{I}S_t - \bar{P}^M_t \, {}^{W}M_t - \bar{P}^S_t \, {}^{W}S_t$$

Equation 20 can be read as "the inventory in period t is equal to the inventory in period (t-1), plus the value of raw materials added to the inventory in period t, plus the value of the subcontracted parts added to the inventory in period t, minus the value of raw materials withdrawn from inventory during period t, minus the value of subcontracted parts withdrawn from inventory during period t."

This formulation not only accounts for the value of stock flowing into and out of inventory at each period, but appreciation or depreciation of stock in inventory.

Plant and Equipment

The equation identifying the value of plant and equipment in the balance sheet is written:

$$(21) \qquad K_t = \sum_{t=1}^{t} \bar{P}_t^K \, \Delta K_t \quad .$$

This equation computes the sum of the original purchase price or "book value" of each item of plant and equipment purchased in period t. The value of plant and equipment is decreased by the last term on the left side of the balance sheet equation, accumulated depreciation.

Accumulated Depreciation

The accumulated depreciation term is written:

$$(22) \qquad \sum_{t=1}^{t} \delta K_t = \sum_{t=1}^{t} P_t^K \, \bar{\delta} K_t$$

Equation 22 can be read as "the total consumed capital is equal to the sum of the depreciated capital in period t, times the cost of replacing that capital in period t." The interpretation of the depreciation rate $\bar{\delta}_t$ is that $\bar{\delta}_t$ represents the physical rate of consumption of capital in the firm.

Liabilities and Net Worth

The right side of the balance sheet and the balance sheet equation represent the liabilities and net worth. Each term is discussed below.

Accounts Payable

The first term under liabilities and net worth is accounts payable Ap_t. The equation describing accounts payable is written:

$$(23) \qquad Ap_t = \bar{P}^M_{t-1} \; I_{M_{t-1}} + \bar{P}^M_t \; I_{M_t} + \bar{P}^S_{t-1} \; I_{S_{t-1}} + \bar{P}^S_t \; I_{S_t}$$

Equation 23 can be read as "the accounts payable in period t is equal to the cost of raw materials received into inventory in period (t-1), plus the cost of raw materials received into inventory in period t, plus the cost of subcontracted parts received into inventory in period (t-1), plus the cost of subcontracted parts received into inventory in period t." This formulation allows the accounts payable to be carried for two periods.

Short Term Debt

The second entry under liabilities and net worth is short term debt. The value of short term debt is written

$$(24) \qquad \mathcal{D}_t = \bar{d} \; D_t$$

which can be read as "the value of short term debt is equal to the funds available to the firm for the insurance of one unit of short term debt, times the number of units of outstanding debt." The equation that accounts for changes in units of short term debt is written:

$$(25) \qquad D_t = D_{t-1} + \Delta D_t - \bar{\phi}_t \; D_t$$

Equation 25 can be read as "the amount of short term debt in period t is equal to the amount of short term debt in period

(t-1), plus the increase in short term debt repaid in period t." Note that ΔD_t is a decision variable to the defense contractor.

Long Term Debt

Long term debt is issued as a bond. Bonds, once issued, are not recalled, thus the amount of long term debt never decreases. The term $\mathcal{B}_t$ in the balance sheet equation represents the value to the firm of all long term debt. The equation which established this value is written

$$(26) \qquad \mathcal{B}_t = \bar{b}\ B_t$$

Equation 26 can be read as "The value of long term debt is equal to the unit value to the firm of one bond, times the number of bonds issued by the firm." The equation that accounts for the number of bonds issued is written:

$$(27) \qquad B_t = B_{t-1} + \Delta B_t$$

Equation 27 can be read as "the number of bonds in period t is equal to the number of bonds in period (t-1), plus the number of bonds issued in period t." Note the issuance of bonds is a decision variable to the firm.

Net Worth

Net worth is the sum of two terms on the balance sheet, equity and retained earnings.

Equity

The equation represented on the balance sheet as equity is written:

$$(28) \qquad \mathcal{E}_t = \Sigma\ \bar{e}_t\ \Delta E_t$$

Equation 28 can be read as "the total value of equity to the firm is equal to the sum of the market selling price of a unit of equity in period t, times the number of shares of equity issued in period t." The issue of allowing a firm to buy its own stock is not considered. The current market value of equity is written $e_t\ E_t$ which can be read as "the market value of equity is the market price of equity in period t, times the number of shares of equity outstanding in period t."

Retained Earnings

The retained earnings term on the balance sheet represents the cumulative retained earnings from time period zero (t=0). Retained earnings must be explained in two equations:

$$(29) \qquad R_t = R_{t-1} + {}^{t}R_t$$

$$(30) \qquad {}^{t}R_t = N_t - \bar{f}_t\ E_t$$

Equation 29 can be read as "the total retained earnings to period t is equal to the total retained earnings to period (t-1), plus the retained earnings from period t." Equation 30 can be read as "the retained earnings in period t is equal to the net income in period t, minus the dividends paid in period t."

Revenue	R_t
Less cost Goods Sold	CGS_t
End Inventories	X_t
Less Beginning Inventories	X_{t-1}
Less Depreciation	δK_t
Gross Profit	GP_t
Less Selling and Administration Costs	$\bar{P}_t^F F_t$
Operating Profit EBIT	$EBIT_t$
Less Interest on Debt	$\bar{\theta} D_t + \bar{\rho} B_t$
EBT	EBT_t
Less Taxes	F(EBT)
Net Income EAT	N_t
Cash Dividends	$f_t E_t$
Retained Earnings	${}^{t}R_t$

Figure 3. INCOME STATEMENT

2. Income Equations

The income statement is a synopsis of the activities of a firm during an accounting period. Figure 3 is an idealized income statement. The majority of the income statement in equation form is the net income equation shown below:

$$(31) \quad N_t = (1-r)[R_t - CGS_t + (X_t - X_{t-1}) - \bar{P}_t^F F_t - \bar{\theta} D_t - \bar{\rho} B_t]$$

Two terms that appear on the income statement do not appear in this net income equation. They are cash dividends on owners equity and retained earnings. These two terms sum to equal net income. The net income is the residual in dollar terms of all the activities of a firm during a period. When viewed in perspective over a number of accounting periods, net income is a measure of the success or failure of the firm. Each term on the right side of the net income equation is now discussed in order of appearance on the income statement.

Revenue

The revenue equation is written:

$$(32) \qquad R_t = \sum_j R_t^j + \sum_i R_t^i$$

The revenue for period t is divided into revenue from government contracts plus revenue from commercial contracts. Government contract revenue is written:

$$(33) \qquad \sum_j R_t^j = \sum_j \pi_{t-1}^j + Pr_{t-1}^j + Z_T^j$$

Equation 33 can be read as "the revenue from all government contracts in period t is equal to the sum of revenues from each contract j, where π_{t-1}^j is any profit payment to the firm from contract j, plus the progress payment on contract j from period (t-1), plus Z_T^j the final cost settlement of all costs

of the contract if contract j ends in period T." Revenue from commercial contracts is written:

$$(34) \qquad \sum_i R_t^i = \sum_i P_t^i \, Q_t^i$$

Equation 34 can be read as "the revenue from all commercial contracts in period t is equal to the sum of all the quantities of commercial products delivered, times the price of those products over all commercial contracts i."

Cost Goods Sold

The cost of goods sold equation is written:

$$(35) \qquad CGS_t = \sum_j CGS_t^j + \sum_i CGS_t^i$$

The cost of goods sold is divided into the cost of government contracts and the cost of commercial contracts. Allowable costs on government contracts exclude some overhead costs and most interest costs. The equation representing the cost to the firm of government contracts is written:

$$(36) \qquad \sum_j CGS_t^j = \sum_j \bar{P}_t \, {}^{W}M_t^j + \bar{P}_t^S \, {}^{W}S_t^j + \bar{P}_t^L \, L_t^j + \bar{P}_r^J \, K_t^j$$

$$+ \bar{P}_t^K \, \delta K_t^j + \bar{P}_t^G \, G_t^j$$

The government cost equation can be read as "the cost of government contracts to the firm in period t is equal to the

sum over all contracts j of the cost of raw materials on contract j in period t, plus the cost of subcontracted parts on contract j in time period t, plus the cost of labor on contract j in time period t, plus the operating capital used on contract j in time period t, plus the cost of GFE used on contract j in time period t." The equation representing the cost to the firm of commercial contracts is written:

$$(37) \quad \sum_i CGS_t^i = \sum_i \bar{P}_t \; {}^{W}M_t^i + \bar{P}_t^S \; {}^{W}S_t^i + \bar{P}_t^L \; L_t^i + \bar{P}_t^J \; K_t^i + \bar{P}_t^K \; \delta K_t^i$$

The commercial contract cost equation can be read similarly to the government contract cost equation. The only distinction to be noted in the formulation of the two equations is that in the commercial contract cost term there is no cost of GFE.

Inventory

The inventory term in the net income equation is written: $(X_t - X_{t-1})$. This term can be read as "the value of inventory in period t, minus the value of inventory in period (t-1)." This inventory term accounts for appreciation or depreciation of inventory from period to period. Appreciation or depreciation of inventory is possible through LIFO inventory accounting techniques (see page 34).

Overhead and Selling Expenses

Direct overhead and some selling expenses are included in cost of goods sold. There are overhead and selling costs,

however, which are not directly attributable to products or contracts. There are also some contractor costs such as interest and education which government contracts disallow as recoverable cost. These nonattributable costs are accounted for in the overhead and selling expenses term $\overline{P}_t^F F_t$.

Interest on Debt

Debt is divided into long term debt and short term debt. Each type has a separate term in the net income equation.

Short term interest payments are described by the expression $\overline{\theta} D_t$. This term can be read as "the interest rate on short term debt times the amount of short term debt in period t."

Long term debt payment is described by the expression: $\overline{\rho} B_t$. This term can be read as "the coupon rate $\overline{\rho}$ (a coupon rate is a periodic payment based on an interest rate determined at the time the bond was issued) times the number of bonds outstanding in period t."

Taxes

Tax is a multiplicative term written $(1-\overline{r})$ where $\overline{r}$ is the tax rate. This formulation allows the direct calculation of net income from earnings before taxes (EBT), see the net income equation, page 43.

Since taxes are usually a fixed percentage of EBT, they have a scale effect on net income.

Net Income

It remains only to be said that net income is the term N_t on the left side of the net income equation. The net income equation is part of the objective function on the maximization process in Chapter III.

F. SUMMARY OF MODEL COMPONENTS

The model couched in financial terms and illustrated through the balance sheet and income statement portrays the interrelationships of supply, manufacturing, demand and finance.

The formulation is described in dollar relationships, yet the full range of production decisions, cost, schedule and performance tradeoffs are considered. Also, the firm's decision on investment in capital stock are considered.

III. ANALYSIS OF THE DEFENSE CONTRACTOR MODEL

This model is developed to give insight into defense contractor decisions. The model is formulated to interpret defense contractor decisions from the contractor's viewpoint. It is assumed that a defense contractor makes decisions so as to maximize his net income over some finite future time horizon. Using these criteria, the following mathematical formulation results:

$$(38) \qquad \text{Max.} \quad \sum_{t=1}^{T} \frac{N_t}{(1+\mu)^t}$$

$$\text{s.t.} \quad g_{n,t} \qquad \begin{array}{l} n = 1,\ldots,26 \\ t = 1,\ldots,T \end{array}$$

Problem 38 is interpreted as, "Maximize the sum, over the time horizon (t=1,...,T), of the present value of net income in each period, subject to the constraints and accounting identities ($g_{n,t}$) formulated in the model specification, Chapter II."

The mathematical technique used to analyze problem 38 is the classical non-linear optimization technique using a Lagrangian and satisfying an associated set of Kuhn-Tucker conditions. For further explanation of this technique see Hadley, Ref. [9]. A generalized formulation of the Lagrangian is written:

(39) $\quad L = \sum_{t=1}^{T} \frac{L_t}{(1+\mu)^t}$

(40) $\quad L_t = N_t + \sum_{n=1}^{26} \lambda_{n,t}\, g_{n,t}$

Equation 39 is interpreted as "the Lagrangian formulation of the problem statement 38 and is equal to the sum of the present value of the Lagrangians L_t over the time horizon (t=1,...,T)." The Lagrangian for each time period equation 39 can be read as "the Langrangian in period t is equal to the net income (objective function) plus the sum of the Lagrange multipliers, times their respective constraints." The constraints in this Lagrangian are composed of the defense contractor model accounting identities and decision variable constraints. The following Lagrange multipliers are restricted to be less than or equal to zero, λ_4, λ_7, λ_{10} . Their associated constraints are inequality constraints (see Figure 4). The remaining Lagrange multipliers are unrestricted in sign. Restrictions on the variables of the model follow:

$0 \leq A, Ap, Ar, C, G, H, I, K, M, S, W, \xi, \gamma, \Gamma$

$0 < B, D, E, J, e$

$0 \leq f, r, \alpha, \beta, \delta, \theta, \phi \leq 1$

The decision variables for the firm are: M, S, L, K, Γ, B, D, E, R. The mathematical formulation of this Lagrangian is presented in Figure 4 which follows on the next three pages.

$$
\begin{aligned}
L_t = (1-r)\Big[& \sum_{j=1}^{y} \pi_{t-1}^{j} + Pr_{t-1}^{j} + Z_T^{j} + \sum_{i=1}^{x} P_t^{i} Q_t^{i} + (X_t - X_{t-1}) \\
& - \Big(\sum_{j=1}^{y} \bar{P}_t^{M} \, {}^{W}M_t^{j} + \bar{P}_t^{S} \, {}^{W}S_t^{j} + \bar{P}_t^{L} L_t^{j} + \bar{P}_t^{J} K_t^{j} + \bar{P}_t^{K} \bar{\delta} K_t^{j} \\
& \qquad + \bar{P}_t^{G} G_t^{j} \Big) \\
& - \Big(\sum_{i=1}^{x} \bar{P}_t^{M} \, {}^{W}M_t^{i} + \bar{P}_t^{S} \, {}^{W}S_t^{i} + P_t^{L} L_t^{i} + \bar{P}_t^{I} K_t^{i} + \bar{P}_i^{K} \, {}^{-}K_t^{i} \Big) \\
& - \bar{P}_t^{F} F_t - \bar{\theta} D_t - \bar{\rho} B_t \Big] \\
& + \sum_{j=1}^{y} \lambda_1^{j} \Big(\pi_t^{j} - [\bar{\alpha}^{j} \bar{C}_t^{j} + \bar{\beta}^{j} (\bar{C}_T^{j} - C_T^{j}) + \Big(\sum_{\ell=1}^{Z} \bar{\gamma}_{\ell} (\Gamma_{\ell} - \bar{\Gamma}_{\ell}) \Big)^{j} \\
& \qquad + \bar{\varepsilon}^{j} \Big(\sum_{t=1}^{t} Q_t^{j} - \sum_{t=1}^{t} \bar{Q}_t^{j} \Big) \\
& + \sum_{j=1}^{y} \lambda_2^{j} \Big(Pr_t^{j} - \bar{\xi}^{j} \sum_{t=1}^{t-1} C_t^{j} + \sum_{t=1}^{t-2} Pr_t^{j} \Big) \\
& + \sum_{j=1}^{y} \lambda_3^{j} \Big(C_T^{j} - \sum_{t=1}^{T} Pr_t^{j} - Z_T^{j} \Big) \\
& + \sum_{j=1}^{y} \lambda_4^{j} \Big(\sum_{t=1}^{T} Q_t^{j} - \sum_{t=1}^{T} \bar{Q}_t^{j} \Big) \\
& + \lambda_5 \big({}^{W}M_t - M({}^{M}L_t, {}^{M}K_t) \big) \\
& + \lambda_6 \big(M_{t-1} + {}^{I}M_t - {}^{W}M_t - M_t \big) \\
& + \lambda_7 \big(M_t - {}^{M}K_t \big) \\
& + \lambda_8 \big({}^{W}S_t - S({}^{S}L_t, {}^{S}K_t) \big)
\end{aligned}
$$

FIGURE 4. Lagrangian Equation

$$+ \lambda_9 \, (S_{t-1} + {}^{I}S_t - {}^{W}S_t - S_t)$$

$$+ \lambda_{10} \, (S_t - {}^{S}K_t)$$

$$+ \lambda_{11} \, (K_t - {}^{M}K_t - {}^{S}K_t - {}^{H}K_t)$$

$$+ \lambda_{12} \, (L_t - {}^{M}L_t - {}^{S}L_t - {}^{H}L_t)$$

$$+ \lambda_{13} \, (K_t - K_{t-1} - \Delta K_t + \overline{\delta} K_t)$$

$$+ \sum_{j=1}^{y} \lambda_{14}^{j} \, (C_t^j - \overline{P}_t^M \, {}^{W}M_t^i - \overline{P}_t^S \, {}^{W}S_t^i - \overline{P}_t^L \, L_t^j - \overline{P}_t^K \, \overline{\delta} K_t^j$$

$$- \overline{P}_t^J \, K_t^j - \overline{P}_t^G \, G_t^j)$$

$$+ \sum_{*=i,j}^{x,y} \lambda_{15}^{*} \, (H^*(Q_t^*, \Gamma_\ell^*, {}^{W}M_t^*, {}^{W}S_t^*, L_t^*, K_t^*, G_t^*))$$

$$+ \lambda_{16} \, (A_t + Ar_t + X_t + \sum_{t=1}^{t} \overline{P}_t^K \, \Delta K_t - \sum_{t=1}^{t} \overline{P}_t^K \, \overline{\delta} K_t - {}^{A}P_t$$

$$- \overline{d}_t \, D_t - \overline{b}_t \, B_t - \sum_{t=1}^{t} e_t \, \Delta E_t - {}^{t}R_t)$$

$$+ \lambda_{17} \, (A_t - A_{t-1} - \sum_{j=1}^{y} (Pr_{t-1}^j + \pi_{t-1}^j + Z_T^j) - \sum_{i=1}^{x} P_t^i \, Q_t^i$$

$$+ \overline{P}_{t-2}^M \, {}^{I}M_{t-2} + \overline{P}_{t-1}^S \, {}^{I}S_{t-1} + \overline{P}_t^L \, L_t + \overline{\theta} D_t + \overline{\rho} B_t$$

$$+ \overline{P}_t^F \, F_t + P_t^K \, \Delta K_t + \overline{P}_t^G \, G_t - \overline{d}_t \, \Delta D_t - \overline{b}_t \, \Delta B_t$$

$$- \overline{e}_t \, \Delta E_t + \overline{f}_t \, E_t)$$

Figure 4 (Continued)

$$+ \lambda_{18} \left({}^{A}r_t - \sum_{j=1}^{y} (Pr^j_{t-1} + \pi^j_{t-1} + Z^j_T) - \sum_{i=1}^{x} P^i_t Q^i_t \right)$$

$$+ \lambda_{19} \left({}^{A}p_t - \sum_{t=t-1}^{t} (\bar{P}^M_t I_{M_t} + \bar{P}^S_t I_{S_t}) \right)$$

$$+ \lambda_{20} (X_t - X_{t-1} - \bar{P}^M_t I_{M_t} - \bar{P}^S_t I_{S_t} + \bar{P}^M_t W_{M_t} + \bar{P}^S_t W_{S_t})$$

$$+ \lambda_{21} (D_t - D_{t-1} - \Delta D_t + \bar{\phi}_t D_t)$$

$$+ \lambda_{22} (B_t - B_{t-1} - \Delta B_t)$$

$$+ \lambda_{23} (E_t - E_{t-1} - \Delta E_t)$$

$$+ \lambda_{24} (R_t - R_{t-1} - {}^{t}R_t)$$

$$+ \lambda_{25} ({}^{t}R_t - N_t + \bar{f}_t E_t)$$

$$+ \lambda_{26} \left(\sum_{j=1}^{y} G^j_t - G_t \right)$$

Figure 4 (Continued)

Given the above mathematical formulation, and assuming the generalized production functions in the model are concave, and the necessary and sufficient conditions for the existence of a maximum are satisfied, the following analysis is conducted. This analysis is performed on an intra-time period basis unless specifically stated otherwise.

A. SUPPLY OF INPUT DECISION RULES

Decision rules for the trade off or rules of substitution between supply of input parameters can be expressed as ratios of the variables to be traded off. Three such trade offs are considered below: the trade off between raw materials and subcontracted parts, the trade off between capital and labor, and the trade off between GFE and capital. The trade off between raw materials and subcontracted parts in this model is written:

$$(41) \qquad \frac{\dfrac{dH^*}{d^W M_t^*}}{\dfrac{dH^*}{d^W S_t^*}} = -\frac{d^W S_t^*}{d^W M_t^*} = \frac{P_t^M}{P_t^S}$$

It is observed that the left hand equality of equation 41 is the definition of the Rate of Technical Substitution (RTS).[2]

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[2] Henderson, J.M. and Quant, R.E., Microeconomic Theory A Mathematical Approach, 2d ed., McGraw-Hill Book Company, p. 59, 1958.

The right hand equality expresses a result familiar to traditional economics, that "the RTS of raw materials for subcontracted parts is equal to the ratio of their prices."

The trade off between capital and labor in this model is written:

$$(42) \qquad \frac{\dfrac{dH^j}{dL_t^j}}{\dfrac{dH^j}{dK_t^j}} = -\frac{dK_t^j}{dL_t^j} = \frac{P_t^L}{P_t^J + \delta P_t^K}$$

The left hand equality of equation 42 is the definition of the RTS of labor for capital. The right hand equality of equation 42 can be read as "the RTS of labor for capital is equal to the ratio of the price of labor to the "price" of capital. Here again the traditional economic result is observed. Note that the "price" of capital is composed of the operating costs of capital P_t^J (plant and equipment), plus the cost of replacing depreciated capital P_t^K. Notice that the nature of the model is such that any adjustment in this decision rule must be vice the labor variable.

The trade off between capital and GFE is written:

$$(43) \qquad \frac{\dfrac{dH^j}{dG_t^S}}{\dfrac{dH^j}{dK_t^S}} = -\frac{dK_t^j}{dG_t^j} = \frac{P_t^G}{P_t^K} - \frac{\lambda_{26,t}^j}{\lambda_{15,t}^j \dfrac{dH^j}{dK_t^j}}$$

The left hand equality of equation 43 is the definition of the Rate of Technical Substitution (RTS) of GFE for capital. The right hand equality of equation 43 can be read as "the Rate of Technical Substitution of GFE for capital is equal to the ratio of the price of GFE to the price of capital, minus the marginal profitability of another unit of GFE per unit of capital utilized, given that there is a production function (technological) constraint on the use of capital." Note that in equation 43, when considering trade offs between variables, trade offs not only involve parameters external to the firm P_t^G , P_t^K , but also involve trade offs internal to the firm $\lambda_{26,t}^j$, $\lambda_{15,t}^j$, $\frac{dH^j}{dK_t^j}$. The ratio of the relative prices $\frac{P_t^G}{P_t^K}$ describe the external trade off. The marginal profitability term $\frac{\lambda_{26,t}^j}{\lambda_{15,t}^j \frac{dH^j}{dK_t^j}}$ describes the internal trade offs the firm must consider when substituting GFE for capital.

B. DEMAND FOR OUTPUT DECISION RULES

The output for this model is a vector quantity and performance characteristics. The decision rule associated with trade offs between quantity and a performance variable is written:

$$(44) \qquad \frac{\dfrac{dH^j}{dQ_t^j}}{\dfrac{dH^j}{dM_\ell^j}} = - \frac{d\Gamma_\ell^j}{dQ_t^j} = \frac{\varepsilon_t}{\gamma_\ell} - \frac{\lambda_{4,t}^j}{\lambda_{1,t}^j \; \gamma_\ell}$$

The left hand equality of equation 44 is the definition of the RTS of quantity produced on contract j in period t, to a performance specification of contract j. The right hand equality of equation 44 can be read as "the RTS of quantity on contract j for performance on contract j is equal to the ratio of the incentive reward for quantity produced to the reward for improving a performance specification, minus the loss due to quality performance output trade offs in the production for optimal levels of input.

C. FINANCIAL DECISION RULES

The relationships of short term debt, long term debt and equity can provide insight into the optimal financial structure of a firm. The optimal relationship between short term debt and long term debt is written:

$$(45) \qquad \frac{d_t}{b_t} = \frac{\lambda_{21}}{\lambda_{22}}$$

Equation 45 can be read as "the ratio of the value of a unit of short term debt to the unit value of long term debt (the

price ratio) is equal to the ratio of the marginal profitability to the firm of an additional dollar of short term debt to the marginal profitability of an additional dollar of long term debt." This type of relationship exists for the ratio of short term debt or long term debt to equity as well. Taken in combination these three ratios characterize the financing relationships that should be established by a firm operating at the optimum.

An inter-time period relationship of two finance variables total net worth and equity can be seen in the following equation.

$$(46) \qquad \frac{\lambda_{17,t}}{\lambda_{23,t}} = \frac{\lambda_{17,t-1}}{\lambda_{23,t-1}}$$

Equation 46 can be read as "the ratio of the marginal profitability of another dollar of net worth in period to to the marginal profitability of another dollar of equity in period t is equal to the ratio of the same quantities in the previous period (t-1)." Equation 46 exhibits the optimal relationship of net worth, which is composed of equity and retained earnings, to equity. This relationship is consistent from time period to time period and characterizes the optimum ratio of equity to retained earnings.

D. COST VERSUS REVENUE

In classic economic models, the optimal utilization of an input factor of production is at the point where the marginal revenue of output from the employment of an additional unit of input is equal to the cost of that additional unit of input. Examples of this traditional concept are observed in equations 47 and 48.

The cost-revenue relationship between quantity and raw materials is written:

$$(47) \qquad -(\lambda^j_{1,t}\ \bar{\varepsilon} - \lambda^j_{4,t})\ \frac{dQ^j_t}{d^W M^j_t} = \lambda^j_{14}\ P^M_t + (1-r)\ P^M_t$$

Equation 47 is interpreted as "the marginal revenue product of an additional unit of product per unit of additional raw materials is equal to the marginal cost of those materials, as valued internal to the firm on a particular contract, plus a term which is the cost of raw materials adjusted for taxes." The cost-revenue relationship between quantity and labor is written:

$$(48) \qquad -(\lambda^j_{1,t}\ \bar{\varepsilon} - \lambda^j_{4,t})\ \frac{dQ^j_t}{dL^j_t} = \lambda_{14,t}\ P^L_t + (1-r)\ P^L_t$$

Equation 48 is interpreted as "the marginal revenue product of an additional unit of additional labor is equal to the marginal cost of labor as valued internal to the firm or

particular contract, plus the cost of labor adjusted for taxes. These cost-revenue equations involve internal parameters λ_1^j, λ_4^j, λ_{14}^j, as well as parameters external to the firm r, P_t^L, P_t^M. The cost of labor and raw materials are determined by the demand and supply in the input markets. $\lambda_{1,t}$ and $\lambda_{4,t}$ are parameters internal to the firm which are influenced by the demand exhibited in the output market. λ_{14} is a parameter internal to the firm which is influenced by the financial market.

The analysis in Chapter III only highlights the relationships between the variables in the model of a representative defense contractor. These highlights serve two purposes, first, they verify that many of the traditional economic concepts can be demonstrated in this complicated model. Second, they point out the interactions between the model segments. These interactions demonstrate inter-market relationships which can not be seen in single market models. For example, the cost versus revenue analysis relates the supply of input, demand for output and the financial markets through decision parameters internal to the firm.

IV. CONCLUSION

A. AREAS FOR FUTURE RESEARCH

The implementation of this model to describe a specific firm in a defense industry is the next logical extension of this model. Explicit interpretations of the interrelationships of the markets in this model can not be made until the generalized production functions are replaced with specific production functions associated with a specific firm or industry.

The scenario for the model in this paper is one of a certain world. Further research is necessary to adapt the model to a world of uncertainty. This can be most easily accomplished by applying state preference theory which allows replication of the current model in each state of nature, thus the criterion would become the sum over the states of nature occurring times the probability of that state of nature occurring. The model is constructed in such a manner that only a little modification is necessary to introduce utility theory, expected value theory, and the world of uncertainty.

B. SUMMARY

A model of a representative defense contractor is formulated. The model is formulated in segments which correspond to the markets in which the defense contractor operates. The supply of input segment describes the input variables in the

model. The manufacturing segment characterizes the defense contractors inventory and production process. The demand for output segment describes the output and contractual relationships of the buyer and the seller. The financial segment relates the balance sheet and the income statement to the structure of the firm. A discussion of decision rules for the behavior of the defense contractor operating at equilibrium is conducted.

LIST OF REFERENCES

1. Aerospace Research Center Aerospace Industries Association of America, Inc., Monopsony: A Fundamental Problem in Government Procurement, May 1973.

2. Baron, D. P., "Incentive Contracts and Competitive Bidding," The American Economic Review, v. 62, p. 384-394, June 1972.

3. Beach, E. F., Economic Models An Exposition, John Wiley and Sons, Inc., 1957.

4. Benavie, A., Mathematical Techniques for Economic Analysis, Prentice-Hall, 1972.

5. Commission on Government Procurement, V. 1,2,3,4, December 1972.

6. Cummins, J., Cost Overruns in Defense Procurement: A Problem of Entrepreneurial Choice Under Uncertainty, Ph.D. Thesis, Northwestern University, Evanston, Illinois, August 1973.

7. Dorfman, R., Samuelson, P. A., and Solow, R. M., Linear Programming and Economic Analysis, Rand Corporation, 1958.

8. Gorgal, J. F., and Kleinfeld, I., The Military-Industrial Firm, Praeger Publishers, 1972.

9. Hadley, G., Nonlinear and Dynamic Programming, Addison-Wesley Publishing Company, Inc., 1964.

10. Henderson, J. M. and Quant, R. E., Microeconomic Theory A Mathematical Approach, 2d ed., McGraw-Hill Book Company, 1958.

11. Hirshleifer, J., Investment, Interest and Capital, Prentice-Hall, 1970.

12. Industry Advisory Council Subcommittee, To Consider Defense Industry Contract Financing, June 1971.

13. Institute for Mathematical Studies in the Social Sciences Report 24, The Theory of the Firm Facing Uncertain Demand, by Leland, H.E., January 1970.

14. Logistics Management Institute, Task 66-7, An Examination of the Foundations of Incentive Contracting, May 1968.

15. McCall, J. J., "Probabilistic Microeconomics," The Bell Journal of Economics and Management Science, v. 2, p. 403-433, Autumn 1971.

16. Myer, J. N., Understanding Financial Statements, The New American Library, Inc., 1964.

17. Niskanen, W. A. Jr., Bureaucracy and Representative Government, Aldine-Atherton, 1971.

18. Sandmo, A., "On the Theory of the Competitive Firm Under Price Uncertainty," The American Economic Review, v. 61, p. 65-73, March 1972.

19. Vickers, D., The Theory of the Firm: Production, Capital, and Finance, McGraw-Hill Book Company, 1968.

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